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A Higher Dimensional Cosmological Model in a Scale-Covariant Theory of Gravitation

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Abstract Kaluza-Klein space-time is considered in the presence of a perfect fluid distribution in the scale-covariant theory of gravitation by Canuto et al. (Phys. Rev. Lett. 39:429, 1977). With the help of special law of variation for Hubble's parameter proposed by Bermann (Nuovo Cimento 74B:182, 1983), a cosmological model in five dimensions with a negative constant deceleration parameter is presented in this theory. Some physical and kinematical properties of the model are also discussed.

Keywords Higher dimensional model · Deceleration parameter · Scale-covariant theory

1 Introduction

In recent years there has been a lot of interest in the study of higher dimensional cosmology because of the underlying idea that the cosmos at its early stage of evolution of the universe might have had a higher dimensional era. Witten [1], Applequist et al. [2], Chudos and Detweiler [3] are some of the authors who have initiated the discussion of higher dimensional cosmological models. These models are believed to be physical relevance possibly at the early times before the universe has undergone compactification transitions. Further, Marciano [4] has suggested that the experimental observation of fundamental constants with varying time could produce the evidence of extra dimensions.

Alternative theories of gravitation have been extensively studied in connection with their cosmological implications. Noteworthy among them are the scalar-tensor theories of gravitation formulated by Brans and Dicke [5], Nordtvedt [6], Sen [7], Sen and Dunn [8] and Saez and Ballester [9]. In all these theories a scalar field φ has been introduced in addition to the familiar general relativistic metric tensor field g_{ij}. In Brans-Dicke theory there exists a variable gravitational parameter G.

Canuto et al. [10] formulated scale-covariant theory of gravitation which is a viable alternative to general relativity [11, 12]. In scale-covariant theory Einstein field equations are

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valid in gravitational units whereas physical quantities are measured in atomic units. The metric tensors in two systems of units are related by a conformal transformation

$$g'_{ij} = \varphi^2(x^k) g_{ij} \tag{1}$$

wherein latin indices take values 1, 2, 3, 4, 5, primes denote gravitational units and unprimed denote atomic quantities. The gauge function φ $(0 < \varphi < \infty)$ in its most general formulation is function of all space-time coordinates. Thus using the conformal transformation of the type given by (1) Canuto et al. [10] transformed the usual Einstein equations in to

$$R_{ij} - \frac{1}{2} R g_{ij} + f_{ij}(\varphi) = -8\pi G(\varphi) T_{ij} + \mu(\varphi) g_{ij} \tag{2}$$

where

$$\varphi^2 f_{ij} = 2\varphi\varphi_{i;j} - 4\varphi_i\varphi_j - g_{ij}(\varphi\varphi^k_{;k} - \varphi^k\varphi_k) \tag{3}$$

Here R_{ij} is Ricci tensor, R the Ricci scalar, μ the cosmological 'constant', G the gravitational 'constant' and T_{ij} the energy momentum tensor. A semicolon denotes covariant derivative and φ_i denotes ordinary derivative with respect to x^i. A particular feature of this theory is no independent equation for φ exists. The possibilities that have been considered for the gauge function φ are [10]

$$\varphi(t) = \left(\frac{t}{t_0}\right)^{\epsilon}, \quad \epsilon = \pm 1, \pm\frac{1}{2} \tag{4}$$

where t_0 is constant. The form

$$\varphi \sim t^{\frac{1}{2}} \tag{5}$$

is the one most favored to fit in observations [13, 14].

Higher dimensional cosmological models in alternative theories of gravitation with perfect fluid source and string source are of recent interest in modern cosmology as they do play an important role in the discussion of the early universe. Reddy et al. [15–17], Reddy and Naidu [18, 19] have discussed higher dimensional cosmological models in several alternative theories of gravitation. In this paper, we study a five dimensional Kaluza-Klein cosmological model with the help of Hubble's special law of variation proposed by Bermann [20]. This study is relevant in view of the recent scenario of accelerating universe.

2 Metric and Field Equations

We consider the five dimensional Kaluza-Klein space-time in the form

$$ds^2 = dt^2 - A^2(t)(dx^2 + dy^2 + dz^2) - B^2(t)d\alpha^2 \tag{6}$$

where the fifth coordinate α is taken to be space-like.

We consider the energy-momentum tensor for perfect fluid source as

$$T^i_j = (\rho + p)u_i u_j - p g_{ij} \tag{7}$$

together with

$$u^i u_i = 1, \qquad u^i u_j = 0 \tag{8}$$

where ρ is the energy density, p is the isotropic pressure and u^i is the four-velocity of the fluid. Here ρ, p and φ are homogeneous functions of cosmic time t.

Using comoving coordinates, the field equations (2) and (3) with the help of (7) and (8) for the metric (6) can be, explicitly, written as

$$\frac{2A_{44}}{A}+\left(\frac{A_4}{A}\right)^2+2\frac{A_4}{A}\frac{B_4}{B}+\frac{B_{44}}{B}-\frac{\varphi_{44}}{\varphi}-\frac{A_4}{A}\frac{\varphi_4}{\varphi}-\frac{B_4}{B}\frac{\varphi_4}{\varphi}+\frac{\varphi_4^2}{\varphi^2}=8\pi G(\varphi)p \tag{9}$$

$$3\left(\frac{A_4}{A}\right)^2+3\frac{A_4}{A}\frac{B_4}{B}+\frac{\varphi_{44}}{\varphi}-3\frac{\varphi_4^2}{\varphi^2}-3\frac{A_4}{A}\frac{\varphi_4}{\varphi}-\frac{B_4}{B}\frac{\varphi_4}{\varphi}=-8\pi\pi G(\varphi)\rho \tag{10}$$

$$3\frac{A_{44}}{A}+3\frac{A_4^2}{A^2}-\frac{\varphi_{44}}{\varphi}-3\frac{A_4}{A}\frac{\varphi_4}{\varphi}+\frac{B_4}{B}\frac{\varphi_4}{\varphi}+\frac{\varphi_4^2}{\varphi^2}=8\pi G(\varphi)p \tag{11}$$

where a subscript 4 after an unknown function denotes differentiation with respect to t.

Also the conservation equation, which is a consequence of the field equations (2) and (3), in this theory, is given by [10]

$$\rho_4+(\rho+p)u^k_{ik}+\rho\frac{(G\varphi)_4}{G\varphi}+3p\frac{\varphi_4}{\varphi}=0 \tag{12}$$

For the metric (6) this takes the form

$$\rho_4+(\rho+p)\left(3\frac{A_4}{A}+\frac{B_4}{B}\right)+\rho\left(\frac{G_4}{g}+\frac{\varphi_4}{\varphi}\right)+3p\frac{\varphi_4}{\varphi}=0 \tag{13}$$

3 Kaluza-Klein Universe

Here we obtain a five dimensional Kaluza-Klein cosmological model by solving the field equations (9)–(13) which are four independent equations in five unknowns A, B, ρ, p and ((13) being consequence of the field equations) the gauge function φ is given by (5).

We use the special law of variation for Hubble's parameter proposed by Bermann [20] that yields constant deceleration models of the universe. The rate at which the expansion of the universe is slowing down is measured by the deceleration parameter. It may be noted, here, that most of the well known models of the universe in Einstein theory and scalar-tensor theories with zero curvature including inflationary models are models with constant deceleration parameter only. Bermann and Gomide [21], Maharaj and Naidoo [22], Beesham [23], Johri and Desikan [24], Singh and Desikan [25] and Reddy et al. [26–28] have studied cosmological models in four dimensions with a constant deceleration parameter. We consider constant deceleration parameter model defined by

$$q=\left[\frac{RR_{44}}{R_4^2}\right]=\text{constant} \tag{14}$$

where $R=(A^3B)^{1/3}$ is the overall scale factor of the universe. Here the constant is taken as negative (i.e. it is an accelerating model of the universe).

The solution of (14) is

$$R=(A^3B)^{1/3}=(at+b)^{1/1+q} \tag{15}$$

where $a \neq 0$ and b are constants of integration. This equation implies that the condition of expansion is $1 + q > 0$.

Equations (9) and (11) yield

$$\frac{A_{44}}{A} + 2\frac{A_4^2}{A^2} - 2\frac{A_4}{A}\frac{B_4}{B} - \frac{B_{44}}{B} - 2\frac{A_4}{A}\frac{\varphi_4}{\varphi} + 2\frac{B_4}{B}\frac{\varphi_4}{\varphi} = 0 \tag{16}$$

which admits an exact solution

$$A = kB \tag{17}$$

where $k > 1$ is a constant.

From (15) and (17) we have

$$\begin{aligned} A &= (at + b)^{3/4(1+q)} \\ B &= (1/k)(at + b)^{3/4(1+q)} \end{aligned} \tag{18}$$

Thus the five dimensional cosmological model in this theory can be written (through a proper choice of the integration constants) as

$$ds^2 = dt^2 - t^{\frac{3}{2(1+q)}}(dx^2 + dy^2 + dz^2) - \frac{1}{k^2} t^{\frac{s}{2(1+q)}} d\alpha^2 \tag{19}$$

4 Physical Discussion

Kaluza-Klein cosmological model in scale-covariant theory of gravitation is represented by (19) which is homogeneous and has no initial singularities. For the model (19), kinematical parameters are

$$\text{Spatial volume: } V^3 = \sqrt{g} = t^{\frac{2}{1+q}} \tag{20}$$

$$\text{Expansion scalar: } \theta = \frac{1}{3}u^i_{;i} = \frac{1}{(1+q)t} \tag{21}$$

$$\text{Shear scalar: } \sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{1}{6[(1+q)t^2]} \tag{22}$$

$$\text{Hubble's parameter: } H = \frac{R_4}{R} = \frac{1}{(1+q)t} \tag{23}$$

In the model (19) the physical parameters are

$$\text{Energy density: } 8\pi G\rho = \frac{8q^2 + 28q - 7}{8[(1+q)t]^2} \tag{24}$$

$$\text{Pressure: } 8\pi Gp = \frac{4q^2 - 16q + 7}{8[(1+q)t]^2} \tag{25}$$

$$\text{Gravitational 'constant': } G(\varphi) = \left(\frac{12q^2 + 16q - 57}{8q^2 + 28q - 7}\right)t \tag{26}$$

$$\text{Scalar field: } \varphi = \varphi_0 t^{\frac{1}{2}}, \quad \varphi_0 = \text{const} \tag{27}$$

At the initial moment $t=0$, the spatial volume is zero while θ, ρ, p, σ, and H diverge and G and φ become zero. Also, for large t, V and G tend to infinity while θ, ρ, p, σ and H become zero. The model does not approach isotropy for large values of t, since $\underset{t\to\infty}{Lt} \frac{\sigma^2}{\theta^2} \neq 0$. However for $k=1$ the model becomes spatially isotropic. The particle horizon exists because

$$\int_{t_0}^{t} \frac{dt}{V^3(t)} = \left(\frac{1+q}{q-2}\right)\left(t^{\frac{q-2}{q+1}}\right)_{t_0}^{t} \tag{28}$$

is a convergent integral.

5 Conclusions

Here we have obtained a five dimensional Kaluza-Klein cosmological model in a scale-covariant theory of gravitation formulated by Canuto et al. [10], using the special law of variation proposed by Bermann [20] for Hubble's parameter. The model is expanding, homogeneous, anisotropic, accelerating and non singular. The model will help in understanding the early stage of the evolution of the universe in the scale covariant theory of gravitation.

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